# ENGR 101
# Intro to Engineering:
# Week 3

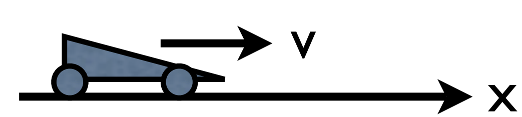
# Section III
# Kinematics

- Motion in one dimension - consider a body moving along an axis

  Since the direction is known we are interested in 4 other quantities: time(t), distance(x), velocity(v), and acceleration(a).

- The kinematic functions or equations are:

  1) $v = v_0 + a\ t$
  2) $x = x_0 + 1/2\ (v_0 + v)\ t$
  3) $x = x_0 + v_0\ t + 1/2\ a\ t^2$
  4) $v^2 = v_0^2 + 2\ a\ (x_0 + x)$



# Equation Derivations

- Equation 1
  $a = \Delta v/\Delta t = (v - v_0)/(t - t_0)$, solving for v yields: $v = v_0 + a\,t$

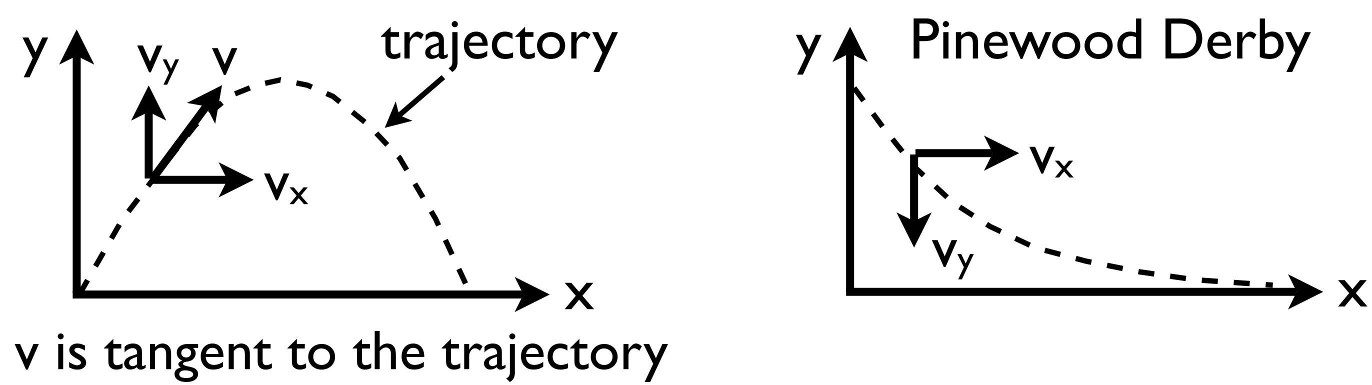
- Equation 2
  $v = \Delta x/\Delta t = (x - x_0)/(t - t_0)$, solve for x but for which v?
  Use the average velocity: $\overline{v} = 1/2\,(v + v_0)$ $x = x_0 + 1/2(v + v_0)t$

- Equation 3
  Substitute 1) into 2): $x = x_0 + 1/2((v_0 + a\,t) + v_0)t$
  $x = x_0 + v_0\,t + 1/2\,a\,t^2$

- Equation 4
  Solve 1) for t: $t = (v - v_0)/a$
  Substitute t into 3) and solve for $v^2$
  $v^2 = v^2_0 + 2\,a\,(x - x_0)$

# Trajectory

- Equations of motion apply in Cartesian systems (x, y, z directions)
- A body in motion in 2 directions describes a trajectory:




- We can use the equations in each direction and combine them using vector mathematics

# Vector Math

- A vector quantity has both a magnitude and a direction. A scalar has only magnitude.

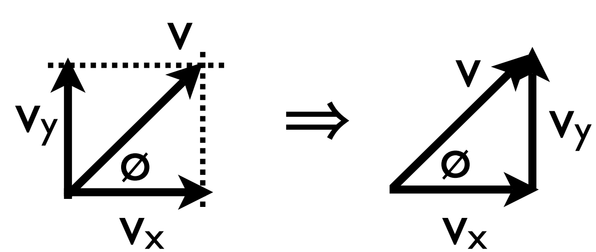
| Vectors | Symbol | Scalars | Symbol |
|---|---|---|---|
| position | **x** | distance | x |
| velocity | **v** | speed | v |
| acceleration | **a** | mass | m |
| force | **F** | temperature | T |

A vector may be 'resolved' into components (x & y components). Combine vectors using Pythagorean Theorem

1) using theorem: $v = (v_x{}^2 + v_y{}^2)^{1/2}$ $\varnothing = \arctan(v_y/v_x)$

2) or using graphics ('triangle law'):

the combination of vectors produces a 'resultant'



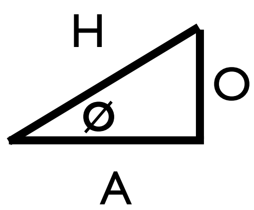

vector addition: add vectors $v_x$ and $v_y$ 'tip to tail'

# More Vectors

- Relations relating to: $\mathbf{v} = \mathbf{v}_x + \mathbf{v}_y$

  1) Vector components: $v_x = v \cos ø$ , $v_y = v \sin ø$

  2) Pythagorean proof of vector addition




$$v^2 = v_x^2 + v_y^2$$
$$v^2 = (v \cos ø)^2 + (v \sin ø)^2$$
$$v^2 = v^2 (\cos ø)^2 + v^2 (\sin ø)^2$$
$$v^2 = v^2 \quad \text{check}$$

  3) $\cos ø = A/H = v_x/v$ $\qquad ø = \cos^{-1}(v_x/v)$
     $\sin ø = O/H = v_y/v$ $\qquad ø = \sin^{-1}(v_y/v)$
     $\tan ø = O/A = v_y/v_x$ $\qquad ø = \tan^{-1}(v_y/v_x)$

- $|| \mathbf{v} ||$ is the magnitude of the vector

# Polynomials & Slope

- Slope = rise/run = $\Delta y/\Delta x$ = $v_y / v_x$ = tan ø
- Polynomial expansion:

  $$y = y(x) \approx \sum_{i=0}^{n} a_i\, x^i = a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + a_n x^n$$

  from the polynomial plot of trajectory y we can derive the slope of the trajectory via:

  $$y' = dy/dx \approx \sum_{i=0}^{n} (i) a_i\, x^{i-1} = 0 + a_1 x^0 + 2a_2 x^1 + \cdots + n a_n x^{n-1}$$

  ex. $y = 3 + 2x^2 + 5x^3 + 0.5x^4 \Rightarrow y' = 4x + 15x^2 + 2x^3$

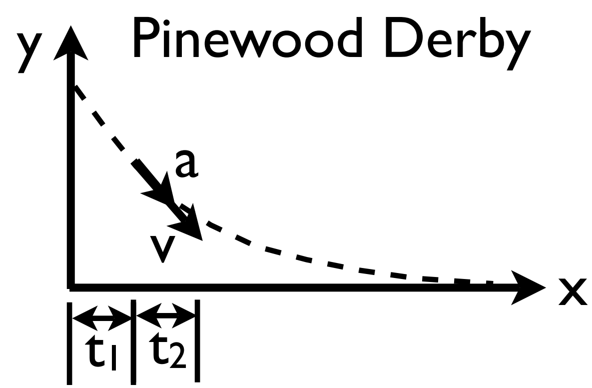
- The Instantaneous Slope, dy/dx is determined by evaluating the limit of $\Delta y/\Delta x$ as $\Delta x \Rightarrow 0$

# Euler's Method

- Consider your Pinewood trajectory. Is the time for the motion of the car, $t = (v_f - v_0)/a$, a good estimate?

- Consider a sequence of time step measurements:
  $t_1 = (v_1 - v_0)/a_1$ , $t_2 = (v_2 - v_1)/a_2$
  $t_{total} = t_1 + t_2$ better approximation

  Continue by extending this approach. We can improve the accuracy: $t_{total} = t_1 + t_2 + ... + t_N$ better yet!




sets of x, y, v, a
e.g. $x_1$, $y_1$, $v_1$, $a_1$,
$x_2$, $y_2$, $v_2$, $a_2$ ...