ENGR 101
Intro to Engineering:
Week 5

Section V: Resistance

- A circuit is composed of two of more devices connected by a conductor

Circuit elements:	Energy type:
emf	source
resistors	sink
capacitors	store
inductors	convert

We will focus on resistance since all devices and conductors oppose the flow of current: they have electrical resistance or impedance ... caused by free electrons colliding with the lattice of atoms of the conductors.

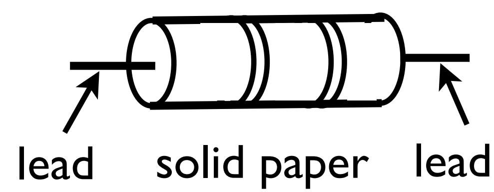
Resistor

- A circuit element specifically for providing resistance is called a <u>resistor</u>. The ratio of the electric pressure $\mathcal{E}$ (voltage) to the current (i):

Georg Ohm
1789-1854
Ohm's Law

$\mathcal{E}$ / i $\equiv$ Volts/Amperes = Ohms [Ω] or [V/A]

resistor
symbol



solid paper
resistor

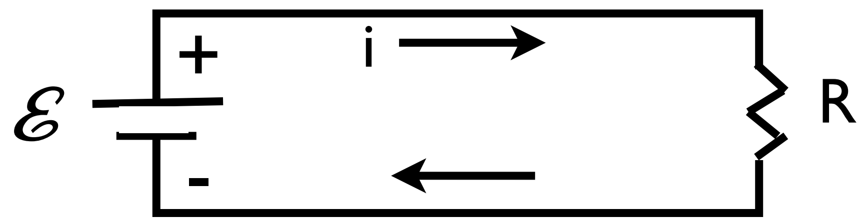
Carbon paper
composite
2.2Ω to 1MΩ

Adjustable
resistors -
potentiometers
rheostats

Resistors absorb electrical energy (sink) and dissipates it as heat. The resistors are rated by their resistances (ohms) and power rating (watts).

Ohm's Law

- Consider the flow of electrons in the following system of conductors or <u>circuit</u>.



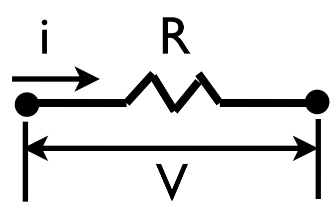
Ohm's Law : the potential difference across a conductor is directly proportional to the current.

Mathematically: $V = R\ I$

constant of proportionality

example:
10 V/10 A = 1Ω

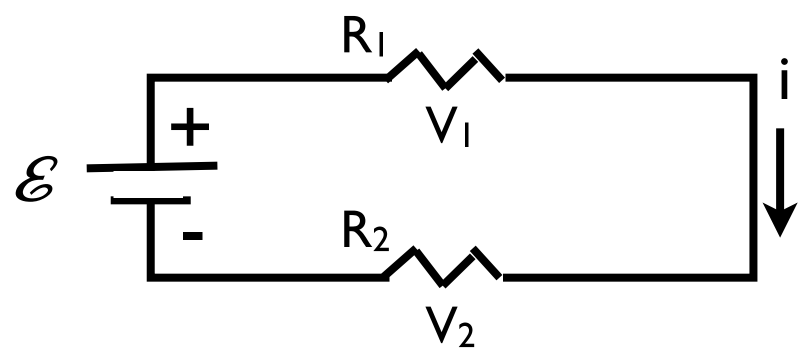
example:
10 V/20 A = 0.5Ω



Need 2 points in the circuit to measure the V - the potential difference

Resistors in Series

- When two or more resistors are connected such that there is only one conducting path between them they are said to be in a series arrangement.



In a series circuit the voltage drop across each resistor is in accordance with Ohm's Law, e.g. $V_1 = - i R_1$ and $V_2 = - i R_2$

Kirchoff's Voltage Law (KVL): the sum of voltage changes around a closed loop is zero. Mathematically: $\sum V = 0$

Equivalent Resistance

- Using Kirchoff's Voltage Law

$$\mathcal{E} - V_1 - V_2 = 0 \quad \text{or} \quad \mathcal{E} = V_1 + V_2$$

- Substituting Ohm's Law

$$\mathcal{E} = i\,R_1 + i\,R_2 = i\,(R_1 + R_2)$$

Dividing by i and applying Ohm's Law again yields the <u>effective</u> or <u>equivalent</u> resistance of the two resistors in series:

$$\mathcal{E}/i = \boxed{R_s = R_1 + R_2}$$

'series resistance'

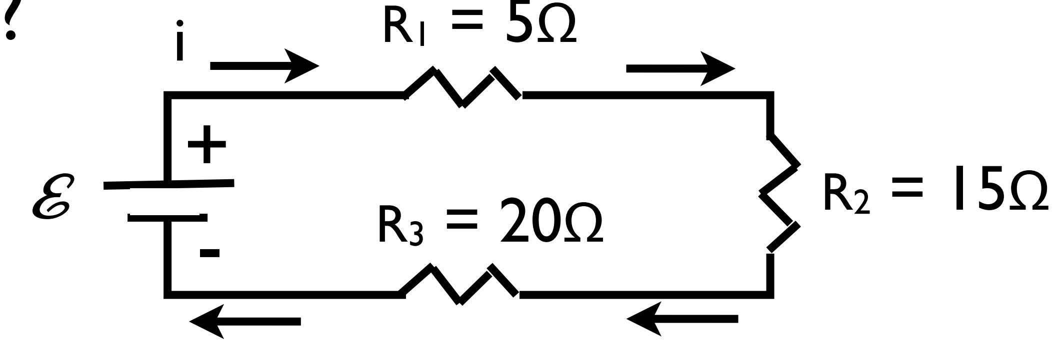
More generally for any number of resistors in series:

$$\boxed{R_s = \sum R}$$

Equivalent Circuit

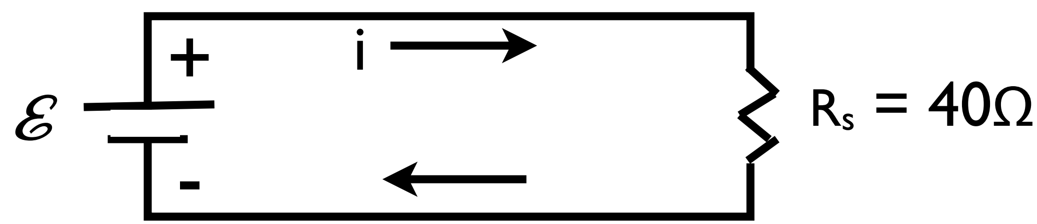
- What is the voltage that must be furnished by the battery to force 0.25A through the following circuit?

Simplify the circuit



The equivalent resistance is: $R_s = \sum R = R_1 + R_2 + R_3 = 5 + 15 + 20 = 40\ \Omega$

The 'equivalent' circuit



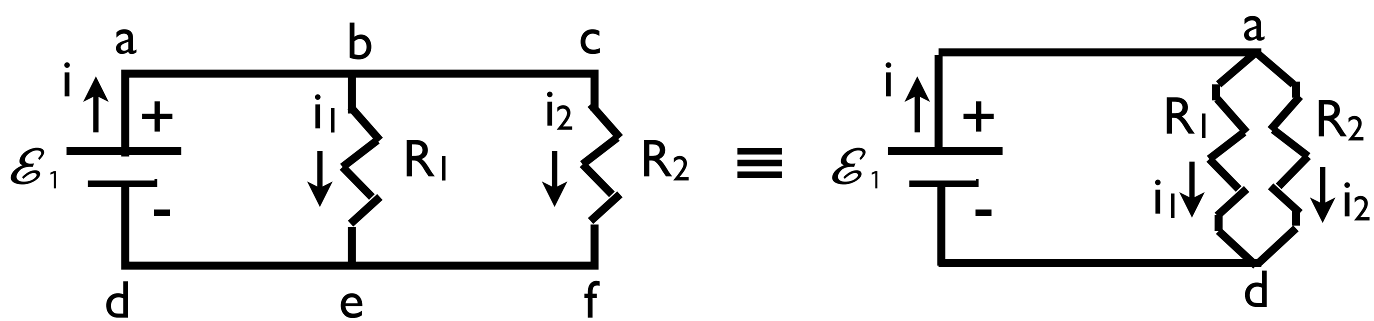
Using KVL and Ohm's Law: $\sum V = 0$ or $\mathcal{E} - V_s = 0$

and $\mathcal{E} - iR_s = 0$, solving for: $\mathcal{E} = 0.25\text{A} \cdot 40\Omega = 10\text{V}$

Resistors in Parallel

- When two or more resistors are connected across the same emf they are said to be in a parallel arrangement.

Simplify the circuit



The 'equivalent' circuit

Def.: A node or junction is a point of connection of two or more circuit elements - a node may not be a single point but a collection of point on a conductor that are electrically identical.

Voltage across resistors in parallel is constant but the current through each resistor is different.

Kirchoff's Current Law

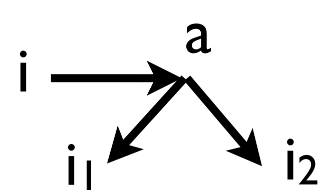
- Kirchoff's Current Law (KCL): requires that the sum of currents at any node sums to zero.

Mathematically: $\boxed{\sum i = 0}$

<u>Convention</u>: current flowing into a junction is positive, current flowing away from a junction is negative.

For the given two resistor circuit, Ohm's Law yields: $i_1 = V_{ad}/R_1$ and $i_2 = V_{ad}/R_2$ where V_{ad} is the potential across nodes a & d. The total current is found by using KCL.

For junction 'a'



$\sum i = i - i_1 - i_2 = 0$

$\boxed{i = i_1 + i_2}$

Equivalent Parallel R

- Apply Ohm's Law to the KCL results:

$$i = i_1 + i_2 = V_{ad}/R_1 + V_{ad}/R_2 = V_{ad}(1/R_1 + 1/R_2)$$

Dividing by V_{ad} and applying Ohm's Law again yields:

$$i/V_{ad} = \boxed{1/R_p = 1/R_1 + 1/R_2}$$

More generally for any number of resistors in parallel:

$$\boxed{1/R_p = \sum 1/R}$$

Invert these formula for R_p

For two resistors in parallel:

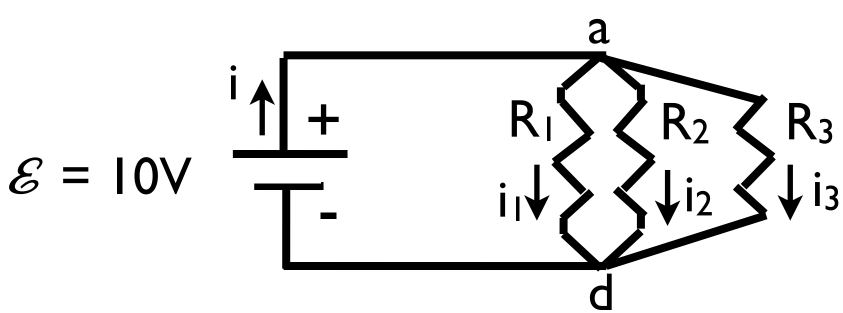
$$1/R_p = (R_1 + R_2)/R_1R_2$$

For three resistors in parallel:

$$1/R_p = [R_3(R_1 + R_2) + R_1R_2]/R_1R_2R_3$$

Examples

- For three resistors connected in parallel



$R_1 = 5\Omega$

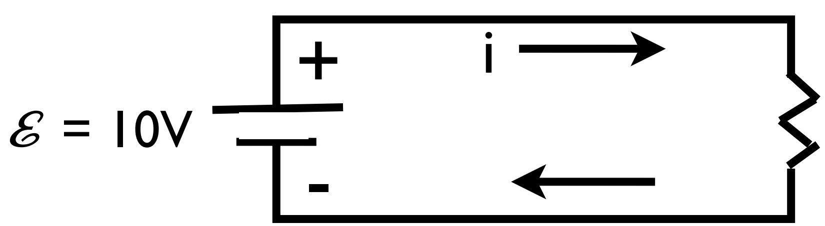
$R_2 = 15\Omega$

$R_3 = 20\Omega$

$1/R_p = \sum 1/R$
$= 1/5 + 1/15 + 1/20$
$= 12/60 + 4/60 + 3/60$
$= 19/60$
$R_p = 60/19 = 3.158\ \Omega$

'parallel' resistance

The 'equivalent' circuit is:

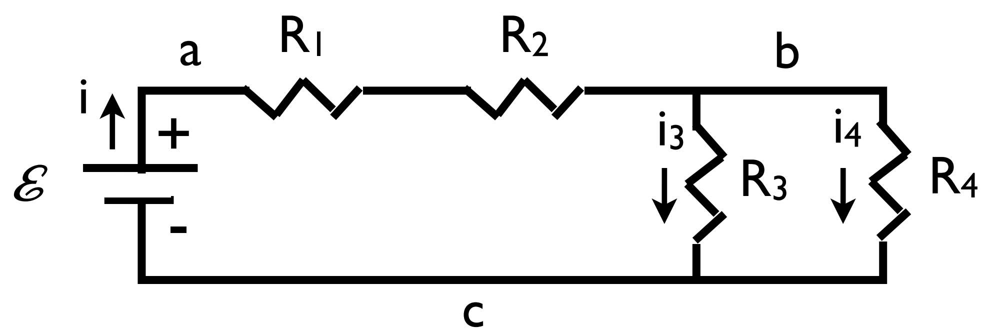


$R_p = 3.158\ \Omega$

Using Ohm's Law: $i = V_{ad}/R_p = 10V / 3.158\ \Omega = 3.167\ A$

Mixed Circuits

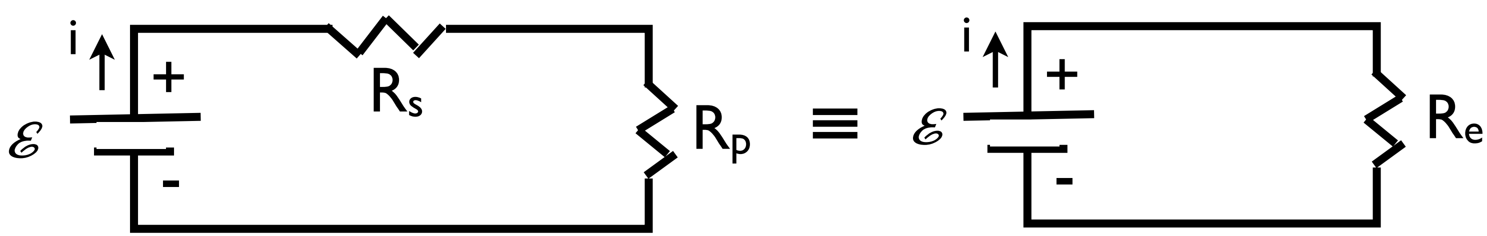
- Series and parallel resistors



Combine R_1 and R_2 in series and R_3 and R_4 in parallel.

$R_s = R_1 + R_2$ | $R_p = R_3 R_4 /(R_3 + R_4)$ | $R_e = R_s + R_p$

'equivalent' resistance



$i = \mathcal{E} / R_e$

$V_s = i R_s$ | $V_p = i R_p$ | $V_1 = i R_1$ | $V_2 = i R_2$ | $i_3 = V_p / R_3$ | $i_4 = V_p / R_4$