BUCKLING of COLUMNS

short or long

$$\sigma = \frac{P}{A}$$

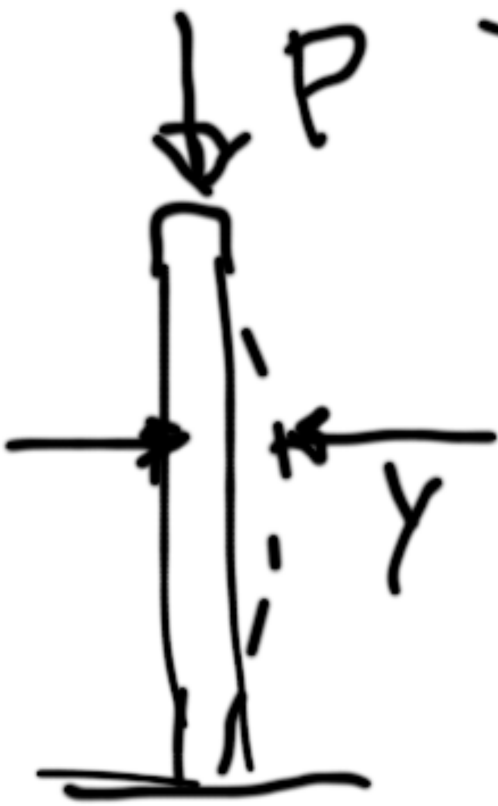
$$\sigma = \frac{P_{crit}}{A}$$

P_{crit} is from a non-linear differential eq.

from $\frac{1}{\rho} = \frac{M}{EI}$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

for a long column



$M = Py$

Substituting

$$\frac{d^2y}{dx^2} = \frac{Py}{EI}$$

a solution is

$$y = A\sin(ax)$$

at $x = L$ $aL = \pi$

$$a = \sqrt{P/EI}$$

also note there needs to be a negative on the Py.

So $\sqrt{P/EI} \cdot L = \pi$

$$\sqrt{P/EI} = \pi/L$$

$$P = \left(\frac{\pi}{L}\right)^2 EI$$

This is the critical load causing buckling.

Euler's formula

Expressing as a critical stress

$$\sigma_{crit} = P_{crit}/A$$

$$\sigma_{crit} = \left(\frac{\pi}{L}\right)^2 \frac{EI}{A}$$

$$r^2 = I/A$$

r is the radius of gyration

then $\sigma_{crit} = \left(\frac{r\pi}{L}\right)^2 E$

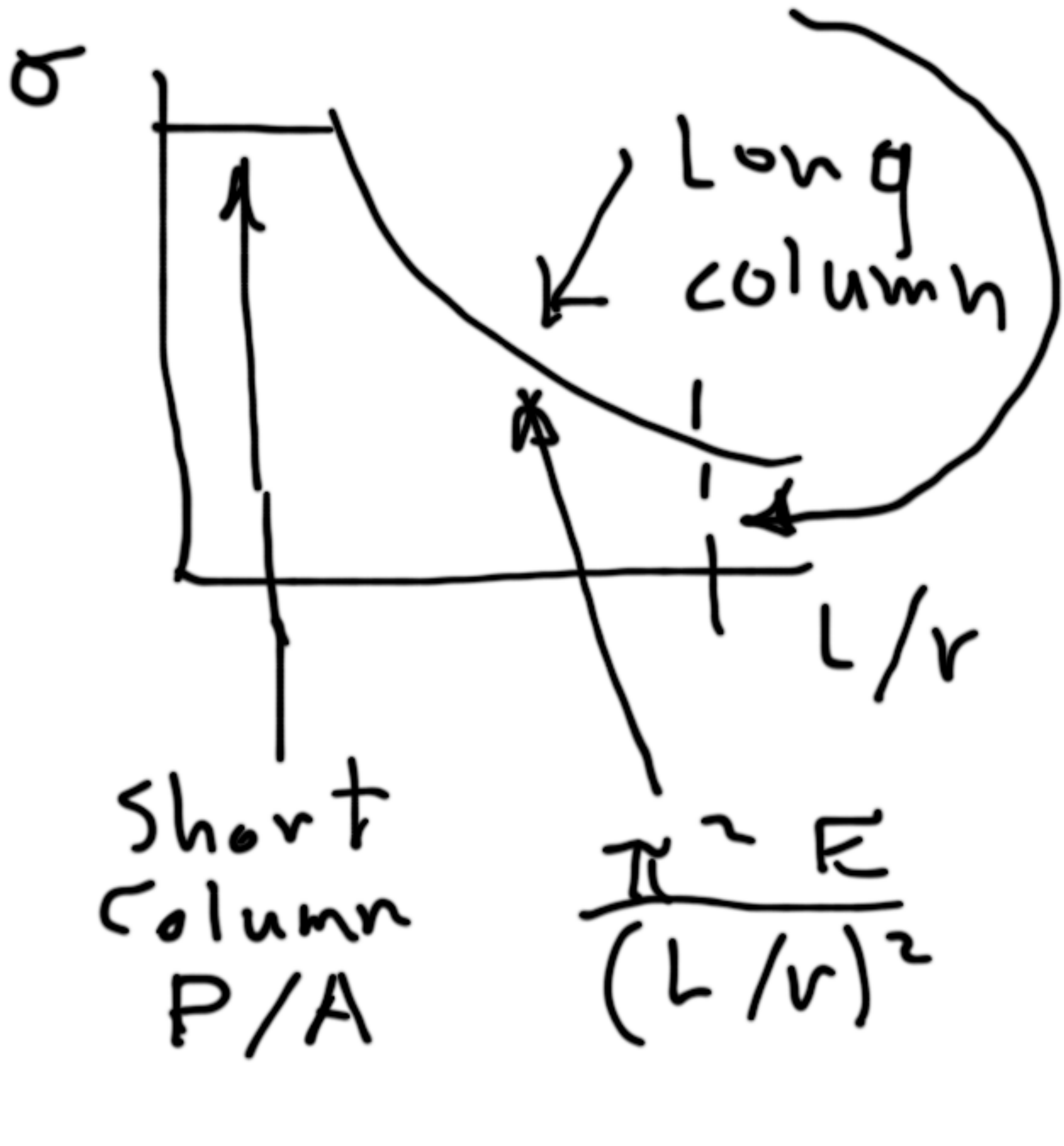
The slenderness ratio is L/r

$$\therefore \sigma_{crit} = \frac{\pi^2 E}{(L/r)^2}$$

Note: L is the effective length which has been multiplied by a factor for different support conditions.

$0.5L \rightarrow 2L$

Structural 'codes' dictate slenderness ratio limitation.



Composite Buckling

P_{crit} is increased due to reinforcement

Use the transformed I^* due to the transformation factor $n = \frac{E_R}{E_m}$

also transform the area by n $\therefore$ A^*