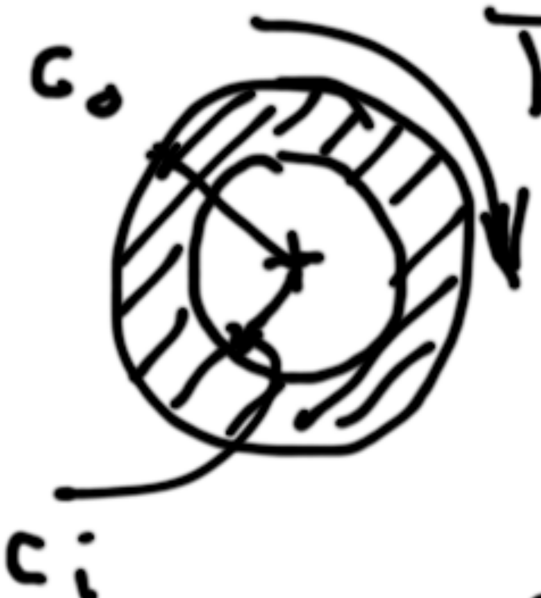
# Torsion




$T \equiv$ torque

$N \cdot m$

$ft \cdot lb$

$$T = \int^{A} \tau \rho \, dA$$

$$= \int_{0}^{c} \tau \rho \, 2\pi\rho \, d\rho$$

but $\tau = \dfrac{\tau_{max}\,\rho}{c}$

from $\dfrac{\gamma}{\rho} = \dfrac{\gamma_{max}}{c}$ & Hooke's Law

So

$$T = \int_0^c \frac{\tau_{max}}{c}\rho \cdot \rho \cdot 2\pi\rho \, d\rho$$

$$= \frac{\tau_{max}}{c}\int_0^c 2\pi\rho^3 \, d\rho$$

$$= \frac{\tau_{max}}{c} \left. \frac{2\pi\rho^4}{4} \right|_0^c$$

$$\tau_{max} = \frac{Tc}{J}$$

where $J \equiv$ polar moment of inertia

$$= \pi c^4 / 2$$

For Hollow Shafts

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$

for the stress at a radius $\rho$ sub $\rho$ for $c$:

$$\tau = \frac{T\rho}{J}$$

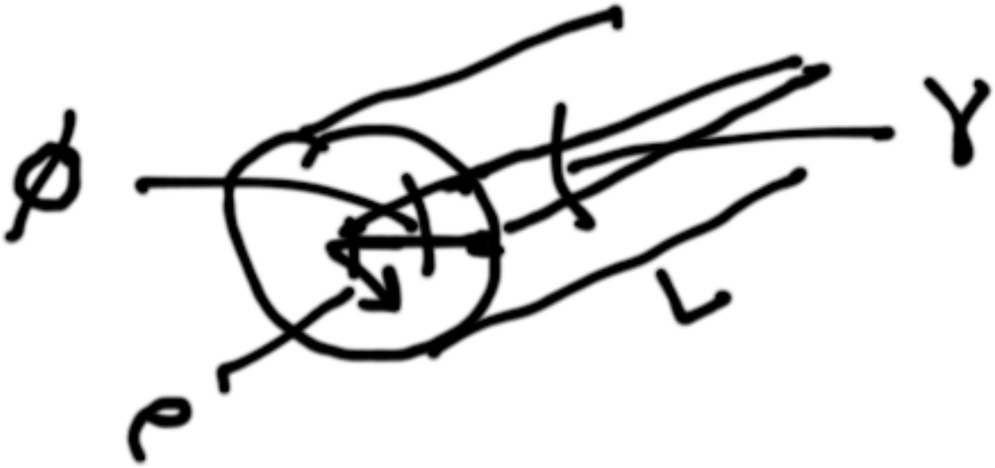
Shear stress increases with $T\uparrow$ and $\rho\uparrow$ and decreases as $J\uparrow$.

At a section $\tau \propto f(\rho)$.

Twist of a shaft is a rotation of concentric rings of a shaft's section.

$\phi$ Twist is caused by torque.

$\gamma$ Strain is rotation of a shaft along it's length.



$\gamma$ vs. $\phi$ relationship

$$\gamma L = \phi \rho$$

using Hooke's Law

$$\tau = G\gamma$$

$$\frac{\tau}{G} L = \phi \rho$$

and $\tau = \frac{T\rho}{J}$

$$\frac{T\rho L}{GJ} = \phi \rho$$

$$\therefore \quad \phi = \frac{TL}{GJ}$$

This proves all the twisted 'elements' are a function of $GJ_s$!

Twist increases as $T \, L \uparrow$ and $GJ \downarrow$.

What are the units of twist?