**Statical Moment of the Area**

for a rectangular section

\[ dQ = y \cdot dA = y \cdot b \, dy \]

the change of \( Q \) is referenced below or above the neutral axis (N.A.)

The integral of \( dQ \) develops the shear stress and the largest \( Q \) develops from the N.A. to the extreme fiber. (The shear stress approaches zero at the extreme fiber.)

1) Integrate the \( Q \) from the N.A to the extreme fiber then, 2) subtract the second integrated \( Q \) from the N.A. to the particular \( y \) layer at which you wish to find the shear stress. This is the net \( Q \).

1) \( \int dQ = \int y \, b \, dy = b \, y^2 /2 \)  

Note: the \( Q(y) \) is a quadratic relationship.

\[ \text{max } Q = b \, d^2 /8 \]

2) \( \int dQ = \int y \, b \, dy = b \, y^2 /2 \)  

for example \( y = d/4 \) (midpoint below N.A.)

mid-point \( Q = b \, d^2 /32 \)

then \( \tau \) at \( y=d/4 \) is

\[ \tau = V \cdot Q(d/2) - Q(d/4) / I \, b = V \cdot b \, d^2 /8 - b \, d^2 /32 / 1/12 \, b \, d^3 \cdot b \]  

= \[ V \cdot 3/32 \, b \, d^2 / 1/12 \, b \, d^3 \cdot b \]  

= \[ V \cdot 36/32 / b d \]  

= \[ 9/8 \, V / b \]

What is the shear stress at \( y=0 \) (at the N.A.)? Substitute into expression for horizontal shear stress to determine max shear stress:

\[ \text{max } \tau = V \, Q / I \, b = V \, b d^2 /8 / 1/12 \, b \, d^3 \cdot b \]

\[ \text{max } \tau = V \, 1/8 / 1/12 \, b \, d = V \, 12 / 8 \cdot b \, d = 3/2 \, V/A \]

You can check these results for the rectangular section by evaluating:

\[ \tau(y) = V \, [ b \, d^2 /8 - b \, y^2 /2 ] / 1/12 \, b \, d^3 \cdot b \]

Generally, \( \tau(y) = V \cdot \text{net } Q(y) / I \cdot b(y) \) and the \( \text{max } \tau \) is where we maximize the numerator and minimize the denominator.